An examination of the sign and volatility switching ARCH models under alternative distributional assumptions

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Abstract

This paper relaxes the assumption of conditional normal innovations used by Fornari and Mele (1997) in modelling the asymmetric reaction of the conditional volatility to the arrival of news. We compare the performance of the Sign and Volatility-Switching ARCH model of Fornari and Mele (1997) and the GJR model of Glosten *et al.* (1993) under the assumption that the innovations follow the Generalised Student's t distribution. Moreover, we hedge against the possibility of misspecification by basing the inferences on the robust variance-covariance matrix suggested by White (1982). The results suggest that using more flexible distributional assumptions on the financial data can have a significant impact on the inferences drawn.

1 Introduction

There is growing evidence that the response of current volatility to past shocks is asymmetric with negative shocks having more impact on current volatility than positive shocks (see Engle and Ng (1993) and Fornari and Mele (1997)). One explanation for this asymmetry is the leverage effect discussed in Black (1976) and Christie (1982). The leverage effect implies that a reduction in the stock price will lead to an increase in the debt to equity ratio measured in terms of market values which might cause an increase in the riskiness of the firm's stocks, and subsequently higher returns' volatility.

In order to model this asymmetry in the response of the conditional volatility to past positive and negative returns shocks, Glosten *et al.* (1993) proposed a model (GJR hereafter) essentially equivalent to the following:

$$\begin{aligned} r_t &= \mu_0 + \mu_1 r_{t-1} + u_t \\ \sigma_t^2 &= \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta_0 u_{t-1}^2 s_{t-1} \end{aligned}$$

where r_t denotes the logarithms of the returns, the innovations u_t follow a distribution with mean 0 and variance σ_t^2 and s_t is the sign of u_t.

Engle and Ng (1993) provided evidence that the GJR model was the best parametric model available at the time to model the asymmetry in the volatility of stock returns.

Fornari and Mele (1997) introduced a more general Sign and Volatility-Switching ARCH model (VS hereafter):

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + (\delta_0 u_{t-1}^2 - \delta_1 \sigma_{t-1}^2 - \delta_2) s_{t-1}$$

which reduces to the GJR model when $\delta_1 = \delta_2 = 0$.

The VS model allows volatility to be influenced not only by the sign of the previous shock, as in GJR model, but also by the size of it. The size of the shock is defined as the unexpected volatility at t-1, given the set of information available at time t-2 ($u_{t-1}^2 - \sigma_{t-1}^2$). A small negative shock at time t-1 which generates lower level of volatility than expected at time t-2 would not lead to a higher volatility at time t, at the same time, a positive shock which produces lower than expected volatility would not lead to a higher volatility at time t. In general, if $\delta_t < 0$, negative shocks generate more volatility than positive ones. When $\delta_t > 0$, positive shocks increase volatility more than negative ones.

Using the fact that the GJR model is nested into the VS model, Fornari and Mele tested whether the MLE estimates of the new coefficients δ_1, δ_2 were significantly different from 0 and found that these coefficients were significantly different from 0 in five out of six data sets (which consisted of daily returns in six international markets: US, Germany, Japan, UK, France, and Italy; the exception was Germany). The tests were performed however under the assumption of normal innovations and without taking into account the possibility of misspecification of the model.

Since normality is not supported by the data, Fornari and Mele (1997) suggested a re-examination of the data, but under more general distributional assumptions. Note that the distributional modelling of financial assets has important consequences for the pricing and hedging options, as recently demonstrated in Pinn (1999).

We undertook this study, generalizing in two directions:

- We used the assumption (suggested by Bollerslev *et al.* (1994)) of a Generalised Student's t distribution for the innovations (This includes the Generalised Error and the Student's t distributions as particular cases).
- Furthemore, we hedge against the possibility of model misspecification by using the robust covariance estimator suggested by White (1982). [1]

[1]Under misspecification, the MLE, now called "quasi" maximum likelihood estimator (QMLE) is to be interpreted as an estimator of the "closest" model from the data in the adopted parametric family (with respect to the Kullback-Leibler distance). However, under misspecification the covariance matrix of the QMLE may no longer be estimated by the inverse of Fisher's information matrix $-\hat{A_n}^{-1}$ (where $A = (\frac{\partial^2}{\partial\theta_i\theta_j} logf(X,\theta))$ and $\hat{A_n}$ denotes averaging with respect to the empirical measure). Instead, as shown by White (1982), it should be estimated by $\hat{A_n}^{-1}\hat{B}_n\hat{A_n}^{-1}$ where B is the outer square of the

Our results indicate that both the distributional assumption and the use of White's robust covariance matrix estimator affect significantly the inference.

As expected, the estimate of the leverage coefficient δ_0 suggested by Glosten *et al.* (1993) has a negative sign and is always significant. However, in contrast with the findings of Fornari and Mele (1997), the estimates of δ_1 and δ_2 are insignificant in the case of US, and marginally significant in the case of Germany (with regard to the other four countries: Japan, UK, France, and Italy, our results are in line with the finding of Fornari and Mele (1997) that the VS model outperforms the GJR model).

Another difference is that the estimates of δ_1 are always positive and the estimates of δ_2 are always negative (Fornari and Mele (1997) also found that δ_1 and δ_2 take opposite signs from each other for each country, but without any pattern emerging across countries).

Our findings are in line with those of other researchers like Bera and Jarque (1982), Baillie and DeGennaro (1990) and Duan (1997) that failure to take into account the distributional properties of stock returns and misspecification may lead to the possibility of wrong inferences being drawn.

Finally, we observe from the continuous limit of the VS model (derived under the more general distributional assumptions, and including a correction to the original derivation), that it may lead to negative volatilities for nonzero values of the parameter δ_2, and as such the model needs to be modified somehow.

The paper is organised as follows. The next section describes the data and methodology. The third section presents the empirical results. Conclusions are given in the fourth section and the continuous limit is derived in the appendix

2 Data and methodology

The data set is the same as that of Fornari and Mele (1997), and was obtained from the Journal of Applied Econometrics Data Archive. The sample has 1494 daily logarithmic returns during the period from 1/1/1990 to 16/10/1995. The series are the S&P 500 (US), Topix (Japan), CAC 40 (France), FTSE 100 (UK), FAZ (Germany) and MIB (Italy).

We use as a probability density function for the innovations u_t the Generalised Student's t distribution, with density:

$$f(u) = \frac{\eta}{2}\frac{\Gamma(\psi+\eta^{-1})}{\Gamma(\psi)\Gamma(\eta^{-1})}k^{-1}\sigma_t^{-1}(1+\frac{|u|^\eta}{\sigma_t^\eta k^\eta})^{-(\psi+\eta^{-1})}$$

where Γ is the gamma function and $k = (\frac{\Gamma(\psi)\Gamma(\eta^{-1})}{\Gamma(3\eta^{-1})\Gamma(\psi-2\eta^{-1})})^{1/2}$.

Note: The constant k equals $b\ \psi^{(1/\eta)}$ in the notation of Bollerslev *et al.* (1994), formula (9.6); however the formula for b there contains a typo.

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gradient $B = (\frac{\partial}{\partial\theta_i}logf(X,\theta)\cdot\frac{\partial}{\partial\theta_j}logf(X,\theta))$ (the classical asymptotic equivalence between B and $-A$ does not hold under misspecification). Also, under misspecification the likelihood ratio test is no longer asymptotically χ^2 distributed and so should preferably be replaced by a Lagrange multiplier or Wald test (see White (1982)).

This class includes the Student's t and Generalised Error densities as special cases, obtained by putting $\eta = 2, \psi = d/2$ (d being the degrees of freedom), and $\psi = \infty$ respectively.

The Student's t distribution for example is obtained by noting that k reduces to $\sqrt{d-2}$ in the case $\eta = 2, \psi = d/2$, yielding:

$$f(u) = \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d}{2})\sqrt{\pi}}(d-2)^{-1/2}\sigma_t^{-1}(1 + \frac{|u|^2}{\sigma_t^2(d-2)})^{(\frac{d+1}{2})}$$

3 Empirical results

The BHHH maximisation routine of Berndt *et al.* (1974) was used to obtain the estimates of the parameters of the VS model using the Generalised Student's t. The SIMPLEX algorithm (see Press *et al.* (1988)) was used to refine the initial values of the parameters finally used as input for the BHHH routine. The parameter estimates reported in Fornari and Mele (1997) were used as starting values for the SIMPLEX algorithm. Different starting values were also tried to ensure that the algorithm reached a global maximum. All our inferences (including the Wald test) are based on the robust covariance estimator proposed by White (1982). For comparison, we estimated the VS and GJR models. The results are in tables 1 and 2.

The most important findings are

1. Comparison with the results of Fornari.

   The results show that the estimates of δ_0 are significant at the 5% level for all six countries under the VS and GJR models. This indicates the existence of significant leverage effects in these markets. An interesting feature of the results is that δ_1 has a positive sign, and δ_2 has a negative sign for all countries. This result is not observed in the Fornari and Mele (1997) paper. The estimates of δ_1 are significant for all countries except for US. This is in contrast to Fornari and Mele (1997) where the estimates of δ_1 are significant for all countries except Germany. The estimates of δ_2 are significant for all countries except US, while they were significant in Fornari and Mele (1997) for US and Japan only. Using the 5% level of significance, and based on the individual t-statistics and the Wald test for the joint hypothesis that $\delta_1 = \delta_2 = 0$, we reject the proposition that the VS model outperforms the GJR model for the US, and we do not reject the same proposition for Germany. The two last conclusions regarding US and Germany are in contrast to the findings of Fornari and Mele (1997). These contradictions between our findings and those of Fornari and Mele (1997) clearly show that using more flexible distributional assumptions can have a significant impact on the inference. [2]

[2] The values of the likelihood for our models are lower than those of Fornari and Mele (1997), even though they should be higher since we are employing a more flexible distribution. Based on attempts to replicate their results assuming the normal distribution, we conjecture that the difference is due to their omitting the constant $-.5Log(2\pi)$ in the log-likelihood function.

Table 1: Parameters of the volatility switching model with the generalized Student's distribution (t-ratios in parentheses)

Parameter/country	U.S.	Germany	Japan	U.K.	France	Italy
μ_0	3.3E-4 (1.99)	1.3E-4 (0.56)	-7.3E-4 (-2.83)	1.08E-4 (30.3)	-1.9E-4 (-0.94)	-2.1E-4 (-0.79)
μ_1	-0.006 (-0.23)	0.045 (1.50)	0.134 (4.78)	0.073 (2.39)	0.03 (1.11)	0.279 (10.47)
ω	7.4E-7 (3.47)	1.2E-6 (2.35)	5.0E-6 (5.18)	1.6E-6 (2.32)	5.9E-6 (15.47)	2.2E-6 (5.66)
α	0.048 (7.45)	0.057 (2.71)	0.15 (15.79)	0.057 (8.22)	0.044 (19.79)	0.073 (19.48)
β	0.942 (275.8)	0.933 (50.12)	0.859 (55.80)	0.922 (51.31)	0.919 (660.8)	0.922 (296.9)
δ_0	-0.027 (-3.34)	-0.045 (-2.36)	-0.097 (-6.86)	-0.033 (-2.68)	-0.044 (-4.92)	-0.026 (-2.38)
δ_1	0.049 (1.21)	0.043 (2.40)	0.194 (10.24)	0.092 (6.37)	0.269 (7.41)	0.15 (8.25)
δ_2	-1.6E-6 (-1.23)	-3.3E-6 (-2.81)	-1.2E-5 (-5.64)	-2.8E-6 (-2.65)	-2.5E-5 (-5.98)	-1.3E-5 (-6.57)
η	1.47 s.e.=0.07	2.45 s.e.=0.05	2.50 s.e.=0.38	3.20 s.e.=0.31	2.58 s.e.=0.16	3.07 s.e.=0.14
ψ	6.14 s.e.=0.67	2.23 s.e.=0.45	1.57 s.e.=0.76	1.39 s.e.=0.35	2.49 s.e.=0.59	1.71 s.e.=0.15
Log Like.	5196.41	4785.22	4371.14	5139.39	4459.04	4372.1
Wald Test $H_0 : \delta_1 = \delta_2 = 0$	1.55	17.42	126.6	40.71	57.77	111.17
SBT	1.56	0.62	0.093	1.43	-0.77	-1.42
NSBT	0.11	1.26	0.027	3.75	1.09	0.62
PSBT	1.09	0.29	0.95	1.06	0.11	-1.19

2. Interesting country specific variation

   In the US case there is no evidence of outperformance for the VS model using tests based either on t-statistics of individual parameters, or on Wald and Likelihood Ratio (LR) tests.[3] In the case of Germany the Wald and LR tests give conflicting results; also, the t-tests on the individual parameters are significant at the 5% level but not the 1% level. The asymmetric behaviour is apparently marginal in the case of Germany. For the remaining countries (excluding US and Germany) our findings support those of Fornari and Mele that the VS model outperforms the GJR model.

3. The rejection of more specific models.

   It is well known that normality is always rejected by the data, which led authors to

[3]The LR test is twice the difference in the likelihood values of the VS and GJR models. If the model is not misspecified, the Wald and LR tests are equivalent, and each of them is asymptotically distributed as χ^2_2. However, the Wald test is preferred to the LR test if misspecification is present.

Table 2: Parameters of the GJR model with the generalized Student's distribution (t-ratios in parentheses)

Par.	U.S.	Germany	Japan	U.K.	France	Italy
μ_0	2.9E-4 (1.99)	1.4E-4 (0.79)	-7.0E-4 (-2.85)	1.4E-4 (0.86)	-1.7E-4 (-0.76)	-1.9E-4 (-0.76)
μ_1	-7.2E-2 (-0.29)	0.044 (1.58)	0.131 (4.71)	0.072 (2.29)	0.027 (1.10)	0.244 (9.16)
ω	0.13E-6 (5.81)	1.1E-6 (32.27)	6.5E-6 (10.79)	2.2E-6 (3.60)	1.2E-5 (5.20)	1.7E-6 (5.27)
α	0.073 (14.5)	0.057 (37.3)	0.17 (18.5)	0.064 (9.82)	0.069 (5.57)	0.058 (19.58)
β	0.909 (214.36)	0.933 (657.02)	0.824 (125.4)	0.899 (75.18)	0.841 (41.4)	0.933 (358.9)
δ_0	-0.04 (-3.79)	-0.043 (-6.86)	-0.12 (-10.2)	-0.039 (-2.94)	-0.062 (-5.47)	-0.027 (-3.07)
η	1.41 s.e.=0.06	2.32 s.e.=0.07	2.5 s.e.=0.35	3.10 s.e.=0.33	2.55 s.e.=0.10	2.79 s.e.=0.13
ψ	6.89 s.e.=1.38	2.55 s.e.=0.16	1.47 s.e.=0.50	1.46 s.e.=0.41	2.47 s.e.=1.5	2.05 s.e.=0.20
Log Like.	5195.08	4784.87	4364.67	5136.3	4445.8	4376.39
SBT	1.79	0.25	0.98	1.84	-0.19	-1.15
NSBT	0.63	1.10	0.56	3.96	1.86	0.21
PSBT	1.10	-0.05	0.92	0.97	0.16	-1.65

propose various more general distributions. For example, the Student's t distribution has been suggested as a candidate for modelling stock returns (see Blattberg and Gonedes (1974)). If we were to apply a t-test for the hypothesis that $\eta = 2$ (the value taken by the Student's t distribution), we would reject this hypothesis for five countries (the exception being Japan) under both the VS and the GJR models.

Finally, the tables report the t-statistics for the Sign Bias Test (SBT), Negative Sign Bias Test (NSBT), and the Positive Sign Bias Test (PSBT), for the residuals from both models, the VS and GJR. These tests were suggested by Engle and Ng (1993) to test for the presence of asymmetric behaviour of volatility. SBT, NSBT, and PSBT are the t-statistics for the coefficients of a linear regression of the squared residuals on S^-_{t-1}, $S^-_{t-1}u_{t-1}$, $S^+_{t-1}u_{t-1}$, respectively, with S^-_t being a dummy variable which equals plus one if $sign(u_t) = -1$ and 0 otherwise, and $S^+_t = 1 - S^-_t$. Fornari and Mele (1997) applied these tests to the original data before estimating their VS model, and found significant asymmetric effects in all series. The same tests were applied to the residuals from the VS and GJR models to check whether there are any asymmetries left. All the tests are not significant at the 5% level of significance, except for the NSBT test for the UK under the VS and GJR models.

4 Conclusions

The paper extended the VS model of Fornari and Mele (1997) by using a more flexible distribution, the Generalised Student's t, and by employing White's robust covariance estimator to draw inferences. Our results support in general the conclusion of Fornari and Mele (1997) that the size of the shock is important in all countries examined except for US. However, country specific contradictions between our results and theirs as well as the discovered patterns in the signs of δ_1, δ_2 illustrate the importance of the more general distributional assumptions.

With regard to the distribution of the innovations of stock returns, the hypothesis that stock returns can be described by the Student's t distribution was rejected for all countries except Japan. It has always been difficult to approximate the distribution of stock returns. Therefore, it is recommended that inferences should be based on the robust covariance matrix estimator as suggested by White (1982).

5 Appendix: The continuous limit of the VS model

We sketch here the derivation of the stochastic differential equation(SDE) satisfied by the variance (or volatility) $v_t = \sigma_t^2$ obtained by taking the continuous limit of the VS model, under the assumption of general innovations with a symmetric distribution and at least four moments. The qualitative structure of the limit will turn out to be the same as that of the VS model, the only change being the appearance of a parameter $c^2 = E(z_t^2 - 1)^2$ in the limit (which is two in the normal case).

Let

$$d\, v_t = \mu(v_t)dt + \nu(v_t)\, dW_t \tag{1}$$

denote the limiting SDE describing the evolution of the volatility. Under appropriate assumptions on μ and ν the volatility process will have a stationary density $p(v)$, given by the well known formula $p(v) = k\frac{e^{s(v)}}{\nu^2(v)}$ where $s(v) = 2\int \frac{\mu}{\nu^2}dv)$ and k is the proportionality constant. It turns out however (after correcting an error in the Fornari and Mele (1997) paper) that when $\delta_2 \neq 0$ the coefficient of the Brownian perturbation $\nu(v)$ in (1) may be strictly positive when $v = 0$; as a consequence of that, the state space of v_t and its stationary distribution are not anymore supported on the positive axis. Thus, v_t is not a proper volatility process and it should be modified somehow (say by replacing it with its absolute value v_t).

Let now h denote the length of the time interval between two observations and let z_t denote the standardised innovations $\frac{u_t}{\sigma_t}$.

The VS recursion for the volatility is given by:

$$v_{t+h} - v_t \;=\; w + (\beta - 1)v_t + \alpha v_t z_t^2 + k_t s_t$$

where $k_t = \delta_0 v_t z_t^2 - \delta_1 v_t - \delta_2$ and s_t is the sign of z_t.

Under appropriate scaling conditions (see the diffusion scaling assumptions below), this difference equation will give rise in the continuous limit $h \to 0$ to a stochastic differential equation (diffusion). The standard method to establish convergence to a diffusion is to show convergence of the first two conditional moments of the increments to the drift and volatility of the diffusion and to show convergence of the conditional higher order cumulants to 0 (for more details see for example Nelson (1990), Duan (1997), or Fornari and Mele (1997)). Below we only sketch the derivation of the drift and variance parameters.

It is convenient to rewrite the difference equation as a sum of terms entirely known at time $t-h$, which will yield the drift and of zero mean "innovations" (terms with conditional expectation 0), each of which will yield in the limit Wiener processes.

$$v_{t+h} - v_t = w + v_t(\beta + \alpha - 1) + \alpha v_t(z_t^2 - 1) + \delta_0 v_t s_t(z_t^2 - 1) + s_t(v_t(\delta_0 - \delta_1) - \delta_2) \tag{2}$$

The terms $z_t^2 - 1, s_t(z_t^2 - 1)$ and s_t above are zero mean innovations. The conditional mean of the increments is thus:

$$\mathbb{E}[(v_{t+h} - v_t)/F_{t-h}] = w + v_t(\beta + \alpha - 1)$$

The conditional variance of the increments is just the sum of the variances, since the symmetry of the Generalised Student's t distribution implies that the innovations $z_t^2 - 1, s_t(z_t^2 - 1), s_t$ are uncorrelated. Thus, the variance is given by

$$Var[(v_{t+h} - v_t)/F_{t-h}] = v_t^2 c^2(\alpha^2 + \delta_0^2) + (v_t(\delta_0 - \delta_1) - \delta_2)^2 \tag{3}$$

where $c^2 = E(z_t^2 - 1)^2$ (under the Generalised Student's t distribution).

To obtain a diffusion in the limit we need to assume that the order of magnitude of the terms which produce the drift is h and that the order of magnitude of the terms which produce the diffusion is $\sqrt{h}$. This requires the following **diffusion scaling assumptions**: $\alpha = \alpha'\sqrt{h}, \delta_0 = \delta_0'\sqrt{h}, \delta_1 = \delta_1'\sqrt{h}, \delta_2 = \delta_2'\sqrt{h}, w = w'h, \alpha + \beta - 1 = -\theta h$, where the primed quantities are asymptotically constant as $h \to 0$ (thus, the original quantities are assumed to be of the order of magnitude $O(h)$ or $O(\sqrt{h})$ respectively).

Under these assumptions, the limiting conditional mean and variance of the increments $v_{t+h} - v_t$ are asymptotically of the form $\mu_t h$ and $\nu_t^2 h$, where

$$\begin{aligned} \mu_t &= w' - \theta v_t \\ \nu_t^2 &= v_t^2 k^2(\alpha')^2 + v_t^2 c^2(\delta_0')^2 + (v_t(\delta_0' - \delta_1') - \delta_2')^2) \end{aligned}$$

Standard diffusion convergence results (see for example Ethier and Kurtz 1986, Theorem 7.4.1) imply that for small h the discrete GARCH volatility will be well approximated by the solution of the SDE:

$$d\ v_t = \mu(v_t)dt + \nu(v_t)\ dW_t \tag{4}$$

where

$$\mu(v) = w' - \theta v \tag{5}$$

$$\nu(v) = \sqrt{v^2k^2(\alpha')^2 + v^2c^2(\delta_0')^2 + (v(\delta_0' - \delta_1') - \delta_2')^2)} \tag{6}$$

Note:

- When $\delta_2' = 0$ we find by plugging in (6) that the SDE (4) reduces to one with linear coefficients

  $$d\ v_t = (w' - \theta v_t)dt + v_t \Phi dW_t \tag{7}$$

  where we put $\Phi^2 = k^2(\alpha')^2 + c^2(\delta_0')^2 + (\delta_0' - \delta_1')^2$. In this well known limiting form of the GJR (and NGARCH) model, the diffusion coefficient $v_t\Phi$ becomes 0 when $v_t = 0$. As a consequence, the state space of v_t may be taken to be the positive numbers.

- When $\delta_2' \neq 0$ the conditional variance is $\nu_t^2 = \Phi^2 v_t^2 + \delta_2^2 - 2d_2' v_t(\delta_0' - \delta_1')$. This corrects the formulas (27)- (28) in Fornari and Mele, who omit the cross term $-2d_2'v_t(\delta_0' - \delta_1')$ (which results when expanding the square in the last term under the square root in (4)). More significantly, in this case the conditional variance ν_t^2 may be strictly positive when $v_t = 0$. As a consequence of this, when $\delta_2' \neq 0$ the Fornari model loses the desirable feature of a positively supported stationary distribution for the volatility of the continuous limit, which is exhibited by the standard GARCH models like GJR, NGARCH and EGARCH and needs to be further modified.

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References

[1] Baillie, Richard T. and Ramon P. DeGennaro (1990), 'Stock Returns and Volatility', Journal of Financial and Quantitative Analysis, 25, 203-214.

[2] Bera, A. K. and Jarque, C. M. (1982), 'Model Specification Tests', Journal of Econometrics, 20, 59-82.

[3] Berndt, E. K., B. H. Hall, R. E. Hall and J. A. Hausman (1974), 'Estimation and Inference in Nonlinear Structural Models', Annals of Economic and Social Measurement, 4, 653- 665.

[4] Black, Fisher (1976), 'Studies of Stock Market Volatility Changes', Proceedings of the American Statistical Association, Business and Economic Statistics Section, 177-181.

[5] Blattberg Robert C., and Nicholas J. Gonedes (1974), 'A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices', Journal of Business, 47, 244-280.

[6] Bollerslev, T., R. Engle and D. Nelson (1994), 'ARCH Models', Ch. 49 in: Engle, R. and D. McFadden (eds.), Handbook of Econometrics, 4, 2959-3038.

[7] Christie, A. A. (1982), 'The Stochastic Behaviour of Common Stock Variances: Value, Leverage and Interest Rate Effects', Journal of Financial Economics, 10, 407-432.

[8] Duan, J. C. (1997), 'Augmented GARCH(p,q) process and its diffusion limit, Journal of Econometrics, 79, 97-127.

[9] Engle, R. F. and V. K. Ng (1993), 'Measuring and Testing the Impact of News on Volatility', Journal of Finance, XLVIII, 5, 1749-1778.

[10] Ethier, S.N. and Kurtz, T.G. (1986), 'Markov processes. Characterisation and convergence, John Wiley, N.Y.

[11] Fornari, F. and Mele, A. (1997), 'Sign- and Volatility-Switching ARCH models: Theory and Applications to International Stock Markets', Journal of Applied Econometrics, 12, 1, 49-65.

[12] Glosten L. R., Ravi Jagannathan and David Runkle (1993), 'On the Relationship between Expected Value and the Volatility of Nominal Excess Return on Stocks', Journal of Finance, XLVIII, 5, 1779-1801.

[13] Nelson, D. (1990), 'ARCH Models as Diffusion Approximations', Journal of Econometrics, 45, 7-38.

[14] Pinn, K. (1999), 'Minimal Variance Hedging of Options with Student-t Underlying', pre-print, http://xxx.lanl.gov/abs/cond-mat/9907421

[15] Press, Flannery, Teukolsky and Vettering (1988), 'Numerical Recipes in C', Cambridge University Press, New York.

[16] White, H. (1982), 'Maximum Likelihood Estimation of Misspecified Models,' Econometrica, 50, 1-25.